Question 1. If K is the curtate future lifetime of (96), calculate $\mathrm{Var}(K)$, given the following life table:

18 De

x	ℓ_x
96	180
97	130
98	73
99	49
100	0

$i = ??$ not giv

Question 2. You are given:

(i) $1000(IA)_{50} = 4996.75$

(ii) $1000A^{\,1}_{50:\overline{1|}} = 5.58$

(iii) $1000A_{51} = 249.05$

(iv) $i = 0.06$

(v) Mortality follows the Illustrative ~~Life Table.~~

1 2 3 4

1 1 1 1

0 1 2 3

$_1E_{50} = .9378$

Calculate $1000(IA)_{51}$.

Question 3. For a select–and–ultimate mortality table with a 3–year select period:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

- Black was a newly selected life on 01/01/2001.
- Black's age on 01/01/2002 is 62.
- Q is the probability on 01/01/2002 that Black will die before 01/01/2006.

Calculate Q.

Question 4. For a special whole life insurance on (40), you are given:

(i) The death benefit is 1000 for the first 10 years and 2500 thereafter.

(ii) Death benefits are payable at the moment of death.

(iii) Z is the present-value random variable.

(iv) Mortality follows De Moivre's law with $\omega = 100$.

(v) $\delta = 0.10$

1000

Calculate $\Pr(Z > 700)$.

Question 5. For a special whole life insurance, you are given:

(i) $b_t = e^{-t}, \quad t > 0$

(ii) μ is constant.

(iii) $\delta = 0.06$

(iv) $Z = e^{-T}v^{T}$, where T is the future lifetime random variable.

(v) $E[Z] = 0.03636$

Calculate $\mathrm{Var}[Z]$.

Question 6. An investment fund is established to provide benefits on 400 independent lives age x.

(i) On January 1, 2001, each life is issued a 10–year deferred whole life insurance of 1000, payable at the moment of death.

(ii) Each life is subject to a constant force of mortality of 0.05.

(iii) The force of interest is 0.07.

Calculate the amount needed in the investment fund on January 1, 2001, so that the probability, as determined by the normal approximation, is 0.95 that the fund will be sufficient to provide these benefits.

Question 7. You are given:

(i) $s(x) = e^{-0.02x}, x \geq 0.$

(ii) $\delta(t) = 0.04, t \geq 0.$

Determine the median of the present value random variable $Z = v^T$ for a whole life policy issued to (x).

Question 8. A 3-year term life insurance to (x) is defined by the following table.

Year	Benefit	k	q_{x+k}
1	3	0	0.20
2	2	1	0.25
3	1	2	0.50

You are given that $v = 0.9$, that death benefits are payable at the end of the year of death and that the expected present value of the death benefit is P. Calculate the probability that the present value of the benefit payment that is actually made will exceed P.

Question 9. For a whole life insurance of 1000 on (x) with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0.04, & 0 < t \leq 10 \\ 0.05, & 10 < t \end{cases}$

(ii) $\mu_x(t) = \begin{cases} 0.06, & 0 < t \leq 10 \\ 0.07, & 10 < t \end{cases}$

Calculate the single benefit premium for this insurance.

ACT 3130 Actuarial Models 1
Test 2 Solution

1.

$$
\begin{aligned}
\mathrm{E}(K) &= \sum_{k=1}^{\infty} {}_kp_x = \frac{\ell_{97}+\ell_{98}+\ell_{99}}{\ell_{96}} = \frac{130+73+49}{180} = 1.4 \\
\mathrm{E}(K^2) &= \sum_{k=1}^{\infty}(2k-1)\,{}_kp_x = \frac{\ell_{97}+3\ell_{98}+5\ell_{99}}{\ell_{96}} = \frac{130+(3)(73)+(5)(49)}{180} = 3.3 \\
\mathrm{Var}(K) &= \mathrm{E}(K^2) - [\mathrm{E}(K)]^2 = 3.3 - (1.4)^2 = 1.34
\end{aligned}
$$

2.

$$
\begin{aligned}
(IA)_{50} &= A^{\;1}_{50:\overline{1}|} + A_{50:\overline{1}|}^{\;\;\;\,1}\,[A_{51} + (IA)_{51}] \\
1000(IA)_{50} &= 1000A^{\;1}_{50:\overline{1}|} + v\,\ell_{51}/\ell_{50}\,[1000A_{51} + 1000(IA)_{51}] \\
4996.75 &= 5.58 + (0.9378)[249.05 + 1000(IA)_{51}] \\
\Rightarrow \quad & (IA)_{51} = 5073
\end{aligned}
$$

3.

$$
\begin{aligned}
Q &= {}_4q_{[61]+1} \\
&= 1 - (p_{[61]+1})(p_{[61]+2})(p_{64})(p_{65}) \\
&= 1 - (0.88)(0.86)(0.84)(0.83) = 0.47236
\end{aligned}
$$

Alternative:

$$
\begin{aligned}
Q &= {}_4q_{[61]+1} \\
&= q_{[61]+1} + (p_{[61]+1})(q_{[61]+2}) + (p_{[61]+1})(p_{[61]+2})(q_{64}) + (p_{[61]+1})(p_{[61]+2})(p_{64})(q_{65}) \\
&= 0.12 + (0.88)(0.14) + (0.88)(0.86)(0.16) + (0.88)(0.86)(0.84)(0.17) = 0.47236
\end{aligned}
$$

4. The present value random variable Z is defined as

$$
Z = \begin{cases} 1000e^{-0.1T} & 0 \le T < 10 \\ 2500e^{-0.1T} & 10 \le T < 60. \end{cases}
$$

Note that Z decreases from 1000 to $1000e^{-0.1(10)} = 367.88$ for $0 \le T < 10$, junps to $2500e^{-0.1(10)} = 919.70$ at $T = 10$, and then decreases to $2500e^{-0.1(60)} = 6.20$ at $T = 60$. In first 10 years,

$$\Pr(Z > 700) = \Pr(1000e^{0.1T} > 700) = \Pr(T < 3.57)$$

After 10 years,

$$\Pr(Z > 700) = \Pr(2500e0.1T > 700) = \Pr(T < 12.73)$$

Thus

$$\Pr(Z > 700) = \Pr(T < 3.57) + \Pr(10 < T < 12.73) = \frac{3.57 + (12.73 - 10)}{60} = 0.105$$

5.

$$\begin{aligned}
\mathrm{E}(Z) &= \int_0^\infty \mathrm{e}^{-t}\mathrm{e}^{-\delta t}\mathrm{e}^{-\mu t}\,\mu\,\mathrm{d}t = \frac{\mu}{\mu+\delta+1} \\
0.03636 &= \frac{\mu}{\mu+0.06+1} \\
\mu &= 0.04 \\
\mathrm{E}(Z^2) &= \int_0^\infty \mathrm{e}^{-2t}\mathrm{e}^{-\delta t}\mathrm{e}^{-\mu t}\,\mu\,\mathrm{d}t = \frac{\mu}{\mu+2\delta+2} = 0.0185 \\
\mathrm{Var}(Z) &= \mathrm{E}(Z^2) - [\mathrm{E}(Z)]^2 = 0.0185 - (0.03636)^2 = 0.0172
\end{aligned}$$

6.

$$\begin{aligned}
{}_{10|}A_x &= v^{10}{}_{10}p_x\frac{\mu}{\mu+\delta} = e^{-(0.07+0.05)10}\frac{0.05}{0.05+0.07} = 0.1255 \\
{}_{10|}{}^2A_x &= v^{20}{}_{10}p_x\frac{\mu}{\mu+2\delta} = e^{-(0.14+0.05)10}\frac{0.05}{0.05+0.14} = 0.0394 \\
\mathrm{E}(S) &= 400 \times 1000 \times 0.1255 = 50200 \\
\mathrm{Var}(S) &= 400 \times 1000^2[0.0394 - (0.1255)^2] = (3075.70)^2 \\
&\Rightarrow \quad F = 50200 + 3075.70 \times 1.645 = 55259.52
\end{aligned}$$

7. Let m be the answer. $0.5 = \Pr[Z \le m] = \Pr[v^T \le m] = \Pr[-T\delta \le \log(m)] = \Pr[T \ge -\log(m)/\delta] = {}_tp_x$ where x is the issue age and $t = -\log(m)/\delta = -25\log(m)$. Since $s(x) = e^{-0.02x}$, then ${}_tp_y = \dfrac{s(y+t)}{s(y)} = e^{-0.02t}$. Hence, $0.5 = {}_tp_x = e^{-0.02[-25\log(m)]} = e^{[(0.5)\log(m)]} = \sqrt{m}$, and $m = (0.5)^2 = 0.25$.
Alternative: We have $F_Z(m) = m^{\mu/\delta} = \sqrt{m} = 0.5$. Thus $m = 0.25$.

8. The first step is to find P. Calculate the present values and the probabilities of paying those present values from the table. Remember that the sum of all ${}_{k|}q_x$'s must be 1. You should obtain the following:

Year	k	Benefit	q_{x+k}	$Z = \mathrm{PV(Benefit)}$	${}_{k\vert}q_x$	${}_{k\vert}q_x \cdot Z$
1	0	3	0.20	2.700	0.2	0.5400
2	1	2	0.25	1.620	0.2	0.3240
3	2	1	0.50	0.729	0.3	0.2187
4+	3+	0		0	0.3	0

Therefore, $P = 1.0827$. The values of Z which are greater that $P = 1.0827$ are $Z = 2.7$ and $Z = 1.620$. Hence $\Pr[Z > P] = 0.2 + 0.2 = 0.4$

9.

$$
\begin{aligned}
1000\overline{A}_x &= 1000[\overline{A}^{\,1}_{x:\overline{10}|} + {}_{10}E_x \overline{A}_{x+10}] \\
&= 1000\left[\int_0^{10} e^{-0.04t}e^{-0.06t}(0.06)\,\mathrm{d}t + e^{-0.4}e^{-0.6}\int_0^{\infty} e^{-0.05t}e^{-0.07t}(0.07)\,\mathrm{d}t\right] \\
&= 1000\left[0.06\int_0^{10} e^{-0.1t}\,\mathrm{d}t + (0.07)e^{-1}\int_0^{\infty} e^{-0.12t}\,\mathrm{d}t\right] \\
&= 1000\left[0.06\left[\frac{-e^{-0.1t}}{0.10}\right]_0^{10} + (0.07)e^{-1}\left[\frac{-e^{-0.12t}}{0.12}\right]_0^{\infty}\right] \\
&\doteq 1000\left[\frac{0.06}{0.1}\left(1-e^{-1}\right) + \frac{0.07}{0.12}e^{-1}\right] = 1000(0.37927 + 0.21460) = 593.87
\end{aligned}
$$

.214596